ISI – Bangalore Center – B Math - Physics III – Mid Term Exam
Date: 11 September 2017. Duration of Exam: 3 hours
Total marks: 70
Answer ALL Questions

Q 1. [Total Marks: 2+4+2+4=12]

a.) Write the two laws of electrostatics in differential form that govern the variation of electric field in space for a given charge distribution $\rho(x)$.

b.) Derive (from one of the above laws) the existence of the electrostatic potential *V*. and find an expression for *V(b)- V(a)* (where *a* and *b* represents two points in space) in terms of the electric field and its integral.

c.) In a certain electrostatic field configuration, the electric field is given by $\vec{E} = \frac{V_0}{R} e^{-r/R} \hat{r}$ where $\hat{r}$ is the unit vector in the radial direction and V_0, R are costants with appropriate units. Calculate the total charge in a sphere of radius r.

d.) For the same problem as in c.) calculate the charge density and the potential.

Q 2. [Total Marks: 10]

A infinitely long cylindrical co-axial cable consists of an inner solid cylinder of radius *a* and an outer thin cylindrical shell of radius *b* (>*a*) . The inner cylinder carries a uniform positive volume charge density ρ and the outer cylindrical thin shell carries a uniform surface charge density. The surface charge is negative and of just the right magnitude so that the cable is electrically neutral.

Show that the potential difference between a point on the axis and a point on the outer cylinder is $-\frac{\rho a^2}{4\varepsilon_0}\left(1+2\ln\left(\frac{b}{a}\right)\right)$.
[Hint: It might be helpful to calculate the electric fields first]

Q 3. [Total Marks: 4+2+4+2=12]

The expression for the total energy of a charge distribution described by a charge density is given by $\frac{1}{2}\int \rho V dv$ where the integral is over the charge distribution and the symbols have their standard meaning. .

a.) Show that for a localized charge distribution the above expression for energy is equal to $\frac{\varepsilon_o}{2}\int(\vec{E}.\vec{E})dv$ where now the integral is over all space.

b.) State explicitly where the assumption for localized charge distribution is used in the above derivation.

c.) For the charge distribution in problem 1.c) calculate the total energy using both $\frac{1}{2}\int \rho V dv$ and $\frac{\varepsilon_o}{2}\int(\vec{E}.\vec{E})dv$ and show that they both are equal to $\frac{1}{2}\varepsilon_0 \pi V_0^2 R$.

d.) Reexamine the argument you have used in part 2a.) and explain the equality of the results in part c.) even though the charge is not localized.

Q 4. [Total Marks: 6+6+4=16]

a.) Let $\rho_1(x)$ and $\rho_2(x)$ be two distinct and unrelated static charge densities with associated potentials $V_1(x)$ and $V_2(x)$ respectively. Prove that

$\int \rho_1 V_2 dv = \int \rho_2 V_1 dv$ where the volume integration is over all space.

b.) Let $\rho(x)$ be the charge density of a electrostatic configuration with *V(x)* the associated potential. Consider a charge free region *R* so that $\rho(x \in R) = 0$. USING THE RESULT OF Q4 a.) , prove the mean value theorem for *V*(x) in region *R*; i,e. prove that the average value of *V* over the surface of a sphere in *R* is equal to the value of *V* at the center of the sphere. Please note that you will not get credit if you prove this result using any method other than the result of part a.) [Hint: You are already given a charge configuration. Consider ANOTHER charge configuration which "picks up" the appropriate values of *V* when you use the result in parts a.)]

c.) Consider an electrostatic configuration in which there are two distinct solid conductors of arbitrary shape with charges on them and no other charge anywhere else. Let *V(x)* be the corresponding potential for this configuration. Let V_1 and V_2 be the potentials on the two conductors. What is the maximum value of *V(x)?* Justify your answer.

Q 5. [Total Marks: 4+4=8]

a.) An electrostatic configuration is such that the electrostatic potential V = constant when restricted to a particular surface. Prove that the electric field at any point on the surface must be normal the surface at that point.

b.) A solid conducting material is in the shape of an ellipsoid bounded by the surface $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, where *a, b, c* are constants. Determine the direction of the electric field at every point on the surface of the conducting material.

Q6. [Total Marks: 2+2+4+4=12]

The behavior of the potential of a localized charge distribution (i.e. no charges at infinity) is given by

$V = V_{mon}(\vec{r}) + V_{dip}(\vec{r}) + O(r^{-3})$, where $V_{mon}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}$ and $V_{dip}(\vec{r}) = \frac{\vec{p}..\vec{r}}{4\pi\varepsilon_0 r^2}$

a.) Give without derivation the expression for Q and $\vec{p}$ in terms of the charge density.

Determine Q and $\vec{p}$ in the following cases (detailed calculation not needed)

b.) for a point charge situated at the origin.

c.) for a charge configuration consists of +q at the origin and a spherical shell of redius R with uniform surface charge density so that the total charge on the shell is –q.

d.) How will the results change if in parts b.) and c.) we use a different coordinate systems so that what was origin is now the point (a,0,0)? Explain your answer.